Variables and Patterns

Introducing Algebra

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How does the number of daylight hours change with the passage of time in a year? Why does this happen?

Who offers the better deal for renting a truck?  
East Coast Trucks: $4.25 for each mile driven or Philadelphia Truck Rental: $200 plus $2 per mile driven.

The group admission price for Wild World Amusement park is $50, plus $10 per person. What equation relates the price to the number of people in the group?
Some things never seem to change. The sun always rises in the east and sets in the west. The United States holds a presidential election every four years. Labor Day always falls on the first Monday of September.

But many other things are always changing. Temperatures rise and fall within a day and from season to season. Store sales change in response to rising and falling prices and shopper demand. Audiences for television shows and movies change as viewers’ interests change. The speeds of cars on streets and highways change in response to variations in traffic density and road conditions.

In mathematics, science, and business, quantities that change are called variables. Many problems require predicting how changes in the values of one variable are related to changes in the values of another. To help you solve such problems, you can represent the relationships between variables using word descriptions, tables, graphs, and equations. The mathematical ideas and skills used to solve such problems come from the branch of mathematics called algebra. This unit introduces some of the basic tools of algebra.
In *Variables and Patterns*, you will study some basic ideas of algebra and learn some ways to use those ideas.

**You will learn how to**
- Identify variables in situations
- Recognize situations in which changes in variables are related in useful patterns
- Describe patterns of change shown in words, tables, and graphs
- Construct tables and graphs to display relationships between variables
- Observe how a change in the relationship between two variables affects the table, graph, and equation
- Use algebraic symbols to write equations relating variables
- Use tables, graphs, and equations to solve problems
- Use graphing calculators to construct tables and graphs of relationships between variables and to answer questions about these relationships

**As you work on problems in this unit, ask yourself questions about problem situations that involve related quantitative variables:**

*What are the variables in the problem?*
*Which variables depend on, or change in relation to, others?*
*How can I use a table, graph, or equation to display and analyze a relationship between quantitative variables?*
*What does it mean when I see regular and predictable changes in a table of data or a graph?*
*How can I use these regular or predictable changes to make estimates or predictions about other data values?*
The bicycle was invented in 1791. People of all ages use bicycles for transportation and sport. Many people spend their vacations taking organized bicycle tours.

RAGBRAI, which stands for Register’s Annual Great Bicycle Ride Across Iowa, is a weeklong cycling tour across the state of Iowa. The event has been held every summer since 1973. Although the tour follows a different route each year, it always begins with as many as 10,000 participants dipping their back bicycle wheels into the Missouri River along Iowa’s western border and ends with the riders dipping their front wheels into the Mississippi River on Iowa’s eastern border.
Sidney, Celia, Liz, Malcolm, and Theo decide to operate bicycle tours as a summer business. The five college students choose a route from Atlantic City, New Jersey, to Norfolk, Virginia. The students name their business Ocean Bike Tours.

While planning their bike tour, the students need to determine how far the touring group can ride each day. To figure this out, they take test rides around their hometowns.

Getting Ready for Problem 1.1

- How far do you think you could ride in a day?
- How do you think the speed of your ride would change during the course of the day?
- What conditions would affect the speed and distance you could ride?

To accurately answer the questions above, you would need to take a test ride yourself. Instead you can perform an experiment involving jumping jacks. This experiment should give you some idea of the patterns commonly seen in tests of endurance.

Jumping Jack Experiment

You will need a group of at least four people:
- a jumper (to do jumping jacks)
- a timer (to keep track of the time)
- a counter (to count jumping jacks)
- a recorder (to write down the number of jumping jacks)

As a group, decide who will do each task.

When the timer says “go,” the jumper begins doing jumping jacks. The jumper continues jumping for 2 minutes. The counter counts the jumping jacks out loud. Every 10 seconds, the timer says “time” and the recorder records the total number of jumping jacks the jumper has done.
Problem 1.1 Interpreting Tables

A. Do the jumping jack experiment. For each jumper, prepare a table for recording the total number of jumping jacks after every 10 seconds, up to a total time of 2 minutes (120 seconds).

| Time (seconds) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | ...
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Number of Jumping Jacks</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use the table of your jumping jack data to answer these questions:

B. How did the jumping jack rates (the number of jumping jacks per second) in your group change as time passed? How is this shown in your tables?

C. What might this pattern suggest about how bike-riding speed would change over a day’s time on the bicycle tour?

ACE Homework starts on page 15.

1.2 Making Graphs

In the jumping jack experiment, the number of jumping jacks and the time are variables. A variable is a quantity that changes or varies. You recorded data for the experiment variables in a table. Another way to display your data is in a coordinate graph. Making a coordinate graph is a way to show the relationships between two variables.
There are four steps to follow when you make a coordinate graph.

**Step 1** Identify two variables.

In Problem 1.1, the two variables are *time* and *number of jumping jacks*.

**Step 2** Select an axis to represent each variable.

Often, you can assign each variable to an axis by thinking about how the variables are related. If one variable depends on the other, put the **dependent variable** on the **y-axis** (the vertical axis) and the **independent variable** on the **x-axis** (the horizontal axis). You may have encountered the terms *dependent variable* and *independent variable* in your science classes.

If time is a variable, you usually put it on the x-axis. This helps you see the “story” that occurs over time as you read the graph from left to right.

In Problem 1.1, the number of jumping jacks depends on time. So, put number of jumping jacks (the dependent variable) on the y-axis and time (the independent variable) on the x-axis.

Label your graph so that someone else can see what it represents. You can label the x-axis as “Time (seconds)” and the y-axis as “Number of Jumping Jacks.” You can use these labels to help you choose a title for your graph. You might title this graph, “Jumping Jacks Over Time.”
Step 3  Select a scale for each axis. For each axis, determine the least and greatest values to show. Then decide how to space the scale marks.

In Problem 1.1, the values for time are between 0 and 120 seconds. On the graph, label the x-axis (time) from 0 to 120. Because you collected data every 10 seconds, label by 10’s.

The scale you use on the y-axis (number of jumping jacks) depends on the number of jumping jacks you did. For example, if you did 97 jumping jacks, you could label your scale from 0 to 100. Because it would take a lot of space to label the scale for every jumping jack, you could label by 10’s.

![Jumping Jacks Over Time Graph](image)

Step 4  Plot the data points.

Suppose that at 60 seconds, you had done 66 jumping jacks. To plot this information, start at 60 on the x-axis (time) and follow a line straight up. On the y-axis (number of jumping jacks), start at 66 and follow a line straight across. Make a point where the two lines intersect. You can describe this point with the coordinate pair (60, 66). The first number in a coordinate pair is the x-coordinate, and the second number is the y-coordinate.

Problem 1.2  Making Graphs

A. Make a graph of the jumping jack data for one of the jumpers in your group.

B. What does your graph show about the jumping jack rate as time passes? (Another way to say this is, what does your graph show about the relationship between the number of jumping jacks and time?)

C. Is the relationship you found between the number of jumping jacks and time easier to see in the table or in the graph? Explain.

ACE  Homework starts on page 15.
Day 1: Atlantic City to Lewes

Sidney, Liz, Celia, Malcolm, and Theo found they could comfortably ride from 60 to 90 miles in one day. They use these findings, as well as a map and campground information, to plan a three-day tour route. They wonder if steep hills and rough winds coming off the ocean might make the trip too difficult for some riders.

It is time to test the projected tour route. The students want the trip to attract middle school students, so Sidney asks her 13-year-old brother, Tony, and her 14-year-old sister, Sarah, to come along. The students will collect data during the trip and use the data to write detailed reports. Using the reports, they can improve their plans and explain the trip to potential customers.

They begin their bike tour in Atlantic City and ride five hours south to Cape May, New Jersey. Sidney and Sarah follow in a van with camping gear. Sarah records distances traveled until they reach Cape May. She makes the table at the right.

From Cape May, they take a ferry across the Delaware Bay to Lewes (LOO-is), Delaware. They camp that night in a state park along the ocean.

### Problem 1.3 Interpreting Graphs

**A.** Make a coordinate graph of the time and distance data in Sarah’s table. Show time on the x-axis.

**B.** Analyze your graph by answering the following questions:

1. Give the coordinate pair for the third point on your graph. What information does this point give?

2. Connecting the points on a graph sometimes helps you see a pattern more clearly. You can connect the points to consider what is happening in the intervals between the points. Connect the points on your graph with straight line segments. Use the line segments to estimate the distance traveled after \( \frac{3}{4} \) of an hour (0.75 hours).
3. The straight-line segment you drew from (4.5, 40) to (5.0, 45) shows the progress if the riders travel at a steady rate for the entire half hour. The actual pace of the group, and of individual riders, may vary throughout the half hour. These paths show some possible ways the ride may have progressed:

i.  

ii.  

iii.  

iv.  

Match each of these connecting paths with the travel notes below.

a. Celia rode slowly at first and gradually increased her speed.

b. Tony and Liz rode quickly and reached the campsite early.

c. Malcolm had to fix a flat tire, so he started late.

d. Theo started off fast. He soon felt tired and slowed down.

C. Sidney wants to describe Day 1 of the tour. Using information from the table or the graph, what can she write about the day’s travel? Consider the following questions:

- How far did the group travel? How much time did it take them?
- During which time interval(s) did they go the greatest distance? During which time interval(s) did they go the least distance?
- Did the riders go farther in the first half or the second half of the day’s ride?

D. Sidney wants to include either the table or the graph in her report. Which do you think she should include? Why?

ACE Homework starts on page 15.
On Day 2, the students leave Lewes, Delaware, and ride through Ocean City, Maryland. They stop for the day on Chincoteague (SHING kuh teeg) Island, which is famous for its annual pony auction.

Assateague (A suh teeg) Island is home to herds of wild ponies. To survive in a harsh environment of beaches, sand dunes, and marshes, these sturdy ponies eat saltmarsh, seaweed, and even poison ivy!

To keep the population of ponies under control, an auction is held every summer. During the famous “Pony Swim,” the ponies that will be sold swim across a quarter mile of water to Chincoteague Island.

Celia collects data along the way and uses it to make the graph below. Her graph shows the distance the riders are from Lewes as the day progresses. This graph is different from the graph made for Problem 1.3, which showed the total distance traveled as Day 1 progressed.
Problem 1.4 Reading Data from Graphs

A. Does it make sense to connect the points on this graph? Explain.

B. Make a table of \((\text{time}, \text{distance})\) data that matches the coordinate pairs of the graph. (You will need to estimate many of the distance values.)

C. What might have happened between hours 2 and 4? What do you think happened between hours 1.5 and 2?

D. During which interval(s) did the riders make the most progress? During which interval(s) did they make the least progress?

E. Which method of displaying the data helps you see the changes better, a table or a graph? Explain.

F. Use the graph to find the total distance the riders travel on Day 2. How did you find your answer?

Homework starts on page 15.

Did You Know?

The Global Positioning System (GPS) is a satellite navigation system funded and operated by the U.S. Department of Defense. However, there are many thousands of civilian users of GPS worldwide. With the use of a portable computer, a Braille keyboard, and a GPS receiver, a blind person is able to get directions.

Go Online For: Information about GPS
PHSchool.com Web Code: ane-9031
Day 3: Chincoteague Island to Norfolk

On Day 3, the group travels from Chincoteague Island to Norfolk, Virginia. Malcolm and Tony ride in the van. They forget to record the distance traveled each half hour, but they do write some notes about the trip.

- We started at 8:30 A.M. and rode into a strong wind until our midmorning break.
- About midmorning, the wind shifted to our backs.
- We stopped for lunch at a barbeque stand and rested for about an hour. By this time, we had traveled about halfway to Norfolk.
- Around 2:00 P.M., we stopped for a brief swim in the ocean.
- Around 3:30 P.M., we reached the north end of the Chesapeake Bay Bridge and Tunnel. We stopped for a few minutes to watch the ships passing. Because riding bikes on the bridge is not allowed, we put the bikes in the van and drove across.
- We took 7.5 hours to complete today’s 80-mile trip.

Problem 1.5 Finding Average Speed

A. Make a table of (time, distance) data that reasonably fits the information in Malcolm and Tony’s notes.

B. Sketch a coordinate graph that shows the same information.

C. Explain how you used each of the six notes to make your table and graph.

D. The riders traveled 80 miles in 7.5 hours. Suppose they had traveled at a constant speed for the entire trip. This constant speed would be the same as the average speed of the real trip. What was the average speed for this trip?

E. Suppose you made a (time, distance) graph for a rider who made the entire 7.5-hour trip traveling at the average speed you found in Question D. What would the graph look like? How would it compare with the graph you made in Question B?

ACE Homework starts on page 15.
Applications

1. A convenience store has been keeping track of its popcorn sales.

<table>
<thead>
<tr>
<th>Time</th>
<th>Total Bags Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>6:00 A.M.</td>
<td>0</td>
</tr>
<tr>
<td>7:00 A.M.</td>
<td>3</td>
</tr>
<tr>
<td>8:00 A.M.</td>
<td>15</td>
</tr>
<tr>
<td>9:00 A.M.</td>
<td>20</td>
</tr>
<tr>
<td>10:00 A.M.</td>
<td>26</td>
</tr>
<tr>
<td>11:00 A.M.</td>
<td>30</td>
</tr>
<tr>
<td>noon</td>
<td>45</td>
</tr>
<tr>
<td>1:00 P.M.</td>
<td>58</td>
</tr>
<tr>
<td>2:00 P.M.</td>
<td>58</td>
</tr>
<tr>
<td>3:00 P.M.</td>
<td>62</td>
</tr>
<tr>
<td>4:00 P.M.</td>
<td>74</td>
</tr>
<tr>
<td>5:00 P.M.</td>
<td>83</td>
</tr>
<tr>
<td>6:00 P.M.</td>
<td>88</td>
</tr>
<tr>
<td>7:00 P.M.</td>
<td>92</td>
</tr>
</tbody>
</table>

**a.** Make a coordinate graph of the data in the table above. Which variable did you put on the x-axis? Why?

**b.** Describe how the number of bags of popcorn sold changed during the day.

**c.** During which hour did the store sell the most popcorn? During which hour did it sell the least popcorn?
2. At the right is a graph of jumping jack data. (On the \(x\)-axis, 20 means the interval from 0 seconds to 20 seconds, 40 means the interval 20 seconds to 40 seconds, and so on.)

   a. What does the graph tell you about Mary’s experiment?
   
   b. How is this graph different from the graph you made in Problem 1.2?
   
   c. What total number of jumping jacks did Mary do?

3. After doing the jumping jack experiment, Andrea and Ken compare their graphs. Because the points on his graph are higher, Ken said he did more jumping jacks in the 120 seconds than Andrea did. Do you agree? Explain.
4. Katrina’s parents kept this record of her growth from her birth until her 18th birthday.

**Katrina’s Height**

<table>
<thead>
<tr>
<th>Age (yr)</th>
<th>Height (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>birth</td>
<td>20</td>
</tr>
<tr>
<td>1</td>
<td>29</td>
</tr>
<tr>
<td>2</td>
<td>33.5</td>
</tr>
<tr>
<td>3</td>
<td>37</td>
</tr>
<tr>
<td>4</td>
<td>39.5</td>
</tr>
<tr>
<td>5</td>
<td>42</td>
</tr>
<tr>
<td>6</td>
<td>45.5</td>
</tr>
<tr>
<td>7</td>
<td>47</td>
</tr>
<tr>
<td>8</td>
<td>49</td>
</tr>
<tr>
<td>9</td>
<td>52</td>
</tr>
<tr>
<td>10</td>
<td>54</td>
</tr>
<tr>
<td>11</td>
<td>56.5</td>
</tr>
<tr>
<td>12</td>
<td>59</td>
</tr>
<tr>
<td>13</td>
<td>61</td>
</tr>
<tr>
<td>14</td>
<td>64</td>
</tr>
<tr>
<td>15</td>
<td>64</td>
</tr>
<tr>
<td>16</td>
<td>64</td>
</tr>
<tr>
<td>17</td>
<td>64.5</td>
</tr>
<tr>
<td>18</td>
<td>64.5</td>
</tr>
</tbody>
</table>

**a.** Make a coordinate graph of Katrina’s height data.

**b.** During which time interval(s) did Katrina have her greatest “growth spurt”?

**c.** During which time interval(s) did Katrina’s height change the least?

**d.** Would it make sense to connect the points on the graph? Why or why not?

**e.** Is it easier to use the table or the graph to answer parts (b) and (c)? Explain.
5. Below is a chart of the water depth in a harbor during a typical 24-hour day. The water level rises and falls with the tides.

**Effect of the Tide on Water Depth**

<table>
<thead>
<tr>
<th>Hours Since Midnight</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (m)</td>
<td>10.1</td>
<td>10.6</td>
<td>11.5</td>
<td>13.2</td>
<td>14.5</td>
<td>15.5</td>
<td>16.2</td>
<td>15.4</td>
<td>14.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hours Since Midnight</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (m)</td>
<td>12.9</td>
<td>11.4</td>
<td>10.3</td>
<td>10.0</td>
<td>10.4</td>
<td>11.4</td>
<td>13.1</td>
<td>14.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hours Since Midnight</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (m)</td>
<td>15.4</td>
<td>16.0</td>
<td>15.6</td>
<td>14.3</td>
<td>13.0</td>
<td>11.6</td>
<td>10.7</td>
<td>10.2</td>
</tr>
</tbody>
</table>

**a.** At what time is the water the deepest? Find the depth at that time.

**b.** At what time is the water the shallowest? Find the depth at that time.

**c.** During what time interval does the depth change most rapidly?

**d.** Make a coordinate graph of the data. Describe the overall pattern you see.

**e.** How did you determine what scale to use for your graph? Do you think everyone in your class used the same scale?
6. Three students made graphs of the population of a town called Huntsville. The break in the y-axis in Graphs A and C indicates that there are values missing between 0 and 8.

**a.** Describe the relationship between time and population as shown in each of the graphs.

**b.** Is it possible that all three graphs correctly represent the population growth in Huntsville? Explain.
7. On the x-axis of the graph below, 6 means the time from 5:00 to 6:00, 7 means the time from 6:00 to 7:00, and so on.

![Juice Vending Machine Sales Graph](image)

a. The graph shows the relationship between two variables. What are the variables?

b. Describe how the number of cans sold changed during the day. Explain why these changes might have occurred.

8. Here is a graph of temperature data collected on the students’ trip from Atlantic City to Lewes.

![Temperatures for Day 1 Graph](image)

a. This graph shows the relationship between two variables. What are they?

b. Make a table of data from this graph.

c. What is the difference between the day’s lowest and highest temperatures?

d. During which time interval(s) did the temperature rise the fastest? During which time interval did it fall the fastest?
e. Is it easier to use the table or the graph to answer part (c)? Why?
f. Is it easier to use the table or the graph to answer part (d)? Why?
g. What information can you get from the lines connecting the points? Do you think it is accurate information? Explain.

9. Here is a graph Celia drew on the bike trip.
a. What does this graph show?
b. Is this a reasonable pattern for the speed of a cyclist? Is this a reasonable pattern for the speed of the van? Is this a reasonable pattern for the speed of the wind? Explain each of your conclusions.

10. Make a table and a graph of \((\text{time}, \text{temperature})\) data that fit the following information about a day on the road:

- **We started riding at 8 A.M.** The day was quite warm, with dark clouds in the sky.
- **About midmorning, the temperature dropped quickly to 63°F, and there was a thunderstorm for about an hour.**
- **After the storm, the sky cleared and there was a warm breeze.**
- **As the day went on, the sun steadily warmed the air.** When we reached our campground at 4 P.M., it was 89°F.

11. When Ben first started to play the electric guitar, his skill increased quite rapidly. Over time, Ben seemed to improve more slowly.
a. Sketch a graph to show how Ben’s guitar-playing skill progressed over time since he began to play.
b. Your graph shows the relationship between two variables. What are those variables?
c. What other variables might affect the rate at which Ben’s playing improves?
12. Amanda made the graphs below to show how her level of hunger and her feelings of happiness changed over the course of a day. She forgot to label the graphs.

Use the following descriptions to determine which graph shows Amanda’s hunger pattern and which graph shows Amanda’s happiness. Explain.

**Hunger:** Amanda woke up really hungry and ate a large breakfast. She was hungry again by lunch, which began at 11:45. After school, she had a snack before basketball practice, but she had a big appetite by the time she got home for dinner. Amanda was full after dinner and did not eat much before she went to bed.

**Happiness:** Amanda woke up in a good mood, but got mad at her older brother for hogging the bathroom. She talked to a boy she likes on the morning bus. Amanda enjoyed her early classes, but got bored by lunch. At lunch, she had fun with friends. She loved her computer class, which was right after lunch, but she didn’t enjoy her other afternoon classes. After school, Amanda had a good time at basketball practice. After dinner, she did homework and chores.
Connections

For Exercises 13–15, order the numbers from least to greatest. Then describe how each number in your ordered list can be obtained from the previous number.

13. 1.75, 0.25, 0.5, 1.5, 2.0, 0.75, 1.25, 1.00
14. \(\frac{3}{8}, \frac{1}{4}, \frac{7}{8}, \frac{1}{8}, \frac{3}{4}, \frac{1}{8}, \frac{5}{8}\)
15. \(\frac{4}{3}, \frac{1}{3}, \frac{4}{8}, \frac{32}{8}, \frac{1}{6}, \frac{3}{13}, \frac{1}{6}, \frac{3}{6}\)

16. Draw the next shape in this pattern. Then, make a table of (number of squares in bottom row, total number of squares) data for the first five shapes in this pattern.

17. Make a table to show how the total number of cubes in these pyramids changes as the width of the base changes from 3 to 7.
18. **Multiple Choice** Suppose you know that there are five blocks in a bag, and one of these is marked “winner.”

You reach into the bag and choose one block at random. What is the probability you will choose the “winner”?

A. $\frac{1}{5}$  
B. $\frac{1}{4}$  
C. $\frac{1}{2}$  
D. None of these

19. a. Suppose you replace the block you chose in Exercise 18 and add another “winner” block. Now there are six blocks in the bag. What is the probability of choosing a “winner” if you choose one block at random?

b. How does your probability of choosing a “winner” change for every extra “winner” block you add to the bag? Use a table or graph to explain your answer.

20. Suppose you toss a 6-sided die twice to make the coordinate pair (roll 1, roll 2). You will win a prize if the result is (2, 2), (4, 4), or (6, 6). What is the probability you will win a prize?

21. The directors of Ocean Bike Tours want to compare their plans with other bicycle tour companies. The bike tour they are planning takes three days, and they wonder if this might be too short. Malcolm called 18 different companies and asked, “How many days is your most popular bike trip?” Here are the answers he received:

3, 6, 7, 5, 10, 7, 4, 2, 3, 3, 5, 14, 5, 7, 12, 4, 3, 6

Make a line plot of the data.

22. **Multiple Choice** What is the median of the data in Exercise 21?

F. 3  
G. 5  
H. 6  
J. 14
23. On the basis of the information in Exercises 21 and 22, should Ocean Bike Tours change the length of the three-day trip? Explain.

24. The graph below shows the results of a survey of people over age 25 who had completed different levels of education.

![Graph showing Education and Salary](image)

a. Make a table that shows the information in the graph.

b. After how many years of education do salaries take a big jump? Why do you think this happens?

c. Do you find it easier to answer part (b) by looking at the graph or at your table? Explain.

25. Think of something in your life that varies with time, and make a graph to show how it might change as time passes. Some possibilities are the length of your hair, your height, your moods, or your feelings toward your friends.
Extensions

26. The number of hours of daylight in a day changes throughout the year. We say that the days are “shorter” in winter and “longer” in summer. The table shows the number of daylight hours in Chicago, Illinois, on a typical day during each month of the year (January is month 1, and so on).

a. Describe any relationships you see between the two variables.

b. On a grid, sketch a coordinate graph of the data. Put months on the x-axis and daylight hours on the y-axis. What patterns do you see?

c. The seasons in the southern hemisphere are the opposite of the seasons in the northern hemisphere. When it is summer in North America, it is winter in Australia. Chicago is about the same distance north of the equator as Melbourne, Australia, is south of the equator. Sketch a graph showing the relationship you would expect to find between the month and the hours of daylight in Melbourne.

<table>
<thead>
<tr>
<th>Month</th>
<th>Daylight Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.0</td>
</tr>
<tr>
<td>2</td>
<td>10.2</td>
</tr>
<tr>
<td>3</td>
<td>11.7</td>
</tr>
<tr>
<td>4</td>
<td>13.1</td>
</tr>
<tr>
<td>5</td>
<td>14.3</td>
</tr>
<tr>
<td>6</td>
<td>15.0</td>
</tr>
<tr>
<td>7</td>
<td>14.5</td>
</tr>
<tr>
<td>8</td>
<td>13.8</td>
</tr>
<tr>
<td>9</td>
<td>12.5</td>
</tr>
<tr>
<td>10</td>
<td>11.0</td>
</tr>
<tr>
<td>11</td>
<td>10.5</td>
</tr>
<tr>
<td>12</td>
<td>10.0</td>
</tr>
</tbody>
</table>

d. Put the (month, daylight) values from your graph in part (c) into a table.
27. Some students did a jumping jack experiment. They reported their data in the graph below.

![Graph of Our Jumping Jack Experiment]

a. According to the graph, how many jumping jacks did the jumper make by the end of 10 seconds? By the end of 20 seconds? By the end of 60 seconds?

b. Give the elapsed time and number of jumping jacks for two other points on the graph.

c. What estimate would make sense for the number of jumping jacks in 30 seconds? The number in 40 seconds? In 50 seconds?

d. What does the overall pattern in the graph show about the rate at which the test jumper completed jumping jacks?

e. Suppose you connected the first and last data points with a straight line segment. Would this line show the overall pattern? Explain.

28. a. A school booster club sells sweatshirts. Which, if any, of the graphs describes the relationship you expect between the price charged for each sweatshirt and the profit? Explain your choice, or draw a new graph you think better describes this relationship.

![Graphs of Profit vs Price]

b. What variables might affect the club’s profits?
29. Chelsea and Nicole can paddle a canoe at a steady rate of 5 miles per hour.

a. On Saturday, they paddle for 3 hours on a calm river. Sketch a graph of their speed over the 3-hour period.

b. On Sunday, they go canoeing again. They paddle with a 2-mile-per-hour current for 1 hour. Then, they turn into a tributary that feeds the river. They paddle against a 2-mile-per-hour current for 2 hours. On the same axes you used in part (a), sketch a graph of their speed over this 3-hour period.

c. How does the speed of the current affect the speed of the canoe?

30. In parts (a)–(e) below, how does the value of one variable change as the value of the other changes? Estimate pairs of values that show the pattern of change you would expect. Record your estimates in a table with at least five data points.

Sample hours of television you watch in a week and your school grade-point average

As television time increases, I expect my grade-point average to decrease.

<table>
<thead>
<tr>
<th>TV Time (hours per week)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade Point Average</td>
<td>3.5</td>
<td>3.25</td>
<td>3.0</td>
<td>2.75</td>
<td>2.5</td>
</tr>
</tbody>
</table>

a. distance from school to your home and time it takes to walk home

b. price of popcorn at a theater and number of bags sold

c. speed of an airplane and time it takes the plane to complete a 500-mile trip

d. number of days you keep a rented DVD and rental charge

e. length of a long-distance telephone call in minutes and cost of the call
The problems in this investigation asked you to think about variables and the patterns relating the values of variables. You made tables and graphs to show how different variables are related. The following questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. Describe the steps you would take in making a graph to show the relationship between two related variables.

2. How do you decide which variable should be on the x-axis and which should be on the y-axis?

3. a. What are the advantages and disadvantages of representing a relationship between variables in a table?
   
   b. What are the advantages and disadvantages of representing a relationship between variables in a graph?
   
   c. What are the advantages and disadvantages of describing a relationship between variables in a written report?
Analyzing Graphs and Tables

In this investigation you will continue to use tables, graphs, and descriptions to compare information and make decisions. Using tables, graphs, and words to represent relationships is an important part of algebra.

Sidney, Celia, Liz, Malcolm, and Theo continue making plans for Ocean Bike Tours. Many of these plans involve questions about money.

- How much will it cost to operate the tours?
- How much should customers pay?
- Will the company make a profit?

The five tour operators decide to do some research.

Getting Ready for Problem 2.1

- With your classmates, make a list of things the tour operators must provide for their customers. Estimate the cost of each item per customer.
- Estimate how much customers would be willing to pay for the three-day tour.
- Based on your estimates, will the partners earn a profit?
Renting Bicycles

The tour operators decide to rent bicycles for their customers. They get information from two bike shops.

Rocky’s Cycle Center sends a table of weekly rental fees for bikes.

**Rocky’s Weekly Rental Rates for Bikes**

<table>
<thead>
<tr>
<th>Number of Bikes</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rental Fee</td>
<td>$400</td>
<td>$535</td>
<td>$655</td>
<td>$770</td>
<td>$875</td>
<td>$1,070</td>
<td>$1,140</td>
<td>$1,180</td>
<td>$1,200</td>
<td></td>
</tr>
</tbody>
</table>

Adrian’s Bike Shop sends a graph of their weekly rental fees. Because the rental fee depends on the number of bikes, they put the number of bikes on the x-axis.

**Problem 2.1 Analyzing a Table and a Graph**

A. Which bike shop should Ocean Bike Tours use? Explain.

B. Suppose you make a graph from the table for Rocky’s Cycle Center. Would it make sense to connect the points? Explain.

C. How much do you think each company charges to rent 32 bikes?

D. 1. What patterns do you find in the table and in the graph?
   2. Based on the patterns you found in part (1), how can you predict values that are not included in the table or graph?

E. 1. Describe a way to find the costs for renting any number of bikes from Adrian’s Bike Shop.
   2. Describe a way to find the costs for renting any number of bikes from Rocky’s Cycle Center.

ACE Homework starts on page 35.
Finding Customers

The tour operators plan a route and choose a bike shop. Now they must figure out what price to charge so they can attract customers and make a profit.

To help set a price, they conduct a survey. They ask 100 people who have taken other bicycle tours which of the following amounts they would pay for the Ocean Bike Tour: $150, $200, $250, $300, $350, $400, $450, $500, $550, or $600. The results are shown in the table below.

Problem 2.2 Making and Analyzing a Graph

A. To make a graph of these data, which variable would you put on the x-axis? Which variable would you put on the y-axis? Explain.

B. Make a coordinate graph of the data on grid paper.

C. Based on your graph, what price do you think the tour operators should charge? Explain.

D. 1. The number of people who say they would take the tour depends on the price. How does the number of potential customers change as the price increases?

2. How is the change in the number of potential customers shown in the table? How is the change shown on the graph?

3. Describe a way to find the number of potential customers for a price between two prices in the table. For example, how can you predict the number of customers for a price of $425?

Price Customers Would Pay

<table>
<thead>
<tr>
<th>Total Price</th>
<th>Number of Customers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$150</td>
<td>76</td>
</tr>
<tr>
<td>$200</td>
<td>74</td>
</tr>
<tr>
<td>$250</td>
<td>71</td>
</tr>
<tr>
<td>$300</td>
<td>65</td>
</tr>
<tr>
<td>$350</td>
<td>59</td>
</tr>
<tr>
<td>$400</td>
<td>49</td>
</tr>
<tr>
<td>$450</td>
<td>38</td>
</tr>
<tr>
<td>$500</td>
<td>26</td>
</tr>
<tr>
<td>$550</td>
<td>14</td>
</tr>
<tr>
<td>$600</td>
<td>0</td>
</tr>
</tbody>
</table>

Ace Homework starts on page 35.
What’s the Story?

It’s important to be good at reading the “story” in a graph. Remember that the y-axis, or vertical axis, of a graph usually represents the dependent variable, and the x-axis, or horizontal axis, represents the independent variable. Here are some questions to ask when you look at a graph.

What are the variables represented by the graph?

Do the values of one variable seem to depend on the values of the other? In other words, do changes in one variable seem to be the result of changes in the other?

What does the shape of the graph say about the relationship between the variables?

Getting Ready for Problem 2.3

The number of cars in a school parking lot changes as time passes during a school day. These graphs show two possibilities for the way the number of cars might change over time.

- Describe the “story” each graph tells about the school parking lot. Which graph shows the pattern you expect?
- How could you label the graph you chose so that someone else would know what it represents?
Questions A–G describe pairs of related variables. For each pair,

- Decide which variable is the dependent variable and which is the independent variable.
- Find a graph that tells a reasonable “story” about how the variables might be related. If no graph tells a reasonable story, sketch your own.
- Explain what the graph tells about the relationship of the variables.
- Give the graph a title.

A. The number of students who go on a school trip is related to the price of the trip for each student.

B. When a skateboard rider goes down one side of a half-pipe ramp and up the other side, her speed changes as time passes.

C. The water level changes over time when someone fills a tub, takes a bath, and empties the tub.

D. The waiting time for a popular ride at an amusement park is related to the number of people in the park.

E. The number of hours of daylight changes over time as the seasons change.

F. Weekly attendance at a popular movie changes as time passes from the date the movie first appears in theaters.

G. The number of customers at an amusement park with water slides is related to the predicted high temperature for the day.
Applications

1. Use the table to answer parts (a)–(e).

<table>
<thead>
<tr>
<th>Age (weeks)</th>
<th>Expected Body Weight (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>birth</td>
<td>1.3</td>
</tr>
<tr>
<td>1</td>
<td>2.3</td>
</tr>
<tr>
<td>2</td>
<td>3.0</td>
</tr>
<tr>
<td>3</td>
<td>3.8</td>
</tr>
<tr>
<td>4</td>
<td>4.5</td>
</tr>
<tr>
<td>5</td>
<td>5.2</td>
</tr>
<tr>
<td>6</td>
<td>6.0</td>
</tr>
<tr>
<td>7</td>
<td>6.7</td>
</tr>
<tr>
<td>8</td>
<td>7.5</td>
</tr>
<tr>
<td>9</td>
<td>7.6</td>
</tr>
<tr>
<td>10</td>
<td>8.9</td>
</tr>
<tr>
<td>11</td>
<td>9.7</td>
</tr>
</tbody>
</table>

a. What weight is predicted for a 1-week-old tiger cub?
b. What weight is predicted for a 10-week-old tiger cub?
c. At what age do tiger cubs typically weigh 7 kilograms?
d. Describe the pattern relating age and weight. Do you expect this pattern to continue indefinitely?
e. Would it make sense to connect the points in a graph of these data?
2. Dezi researches DVD rental prices at local video stores. Source Video has a yearly membership package. The manager gives Dezi this table:

**Source Video Membership/Rental Packages**

<table>
<thead>
<tr>
<th>Number of DVDs Rented</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Cost</strong></td>
<td>$30</td>
<td>$35</td>
<td>$40</td>
<td>$45</td>
<td>$50</td>
<td>$55</td>
<td>$60</td>
</tr>
</tbody>
</table>

Supreme Video does not have membership packages. Dezi makes the graph below to relate the cost at Supreme Video to the number of DVDs rented.

**a.** Both video stores have a good selection of movies. Dezi’s family plans to watch about two movies a month. Which video store should they choose?

**b.** Write a paragraph explaining to Dezi how he can decide which video store to use.

**c.** For each store, describe the pattern of change relating the number of DVDs rented to the cost.

3. The table shows the fees charged at one of the campgrounds on the Ocean Bike Tour.

**Campground Fees**

<table>
<thead>
<tr>
<th>Number of Campsites</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Campground Fee</strong></td>
<td>$12.50</td>
<td>$25.00</td>
<td>$37.50</td>
<td>$50.00</td>
<td>$62.50</td>
<td>$75.00</td>
<td>$87.50</td>
<td>$100.00</td>
</tr>
</tbody>
</table>

**a.** Make a coordinate graph of the data.

**b.** Does it make sense to connect the points on your graph? Explain.

**c.** Using the table, describe the pattern of change in the total campground fee as the number of campsites increases.

**d.** How is the pattern you described in part (c) shown in your graph?
4. Some class officers want to sell T-shirts to raise funds for a class trip. They ask the students in their class how much they would pay for a shirt and record the data in a table.

**Projected Shirt Sales**

<table>
<thead>
<tr>
<th>Price per Shirt</th>
<th>$5</th>
<th>$10</th>
<th>$15</th>
<th>$20</th>
<th>$25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Shirt Sales</td>
<td>50</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Value of Shirt Sales</td>
<td>$250</td>
<td>$400</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**a.** Describe the relationship between the price per shirt and the expected number of shirt sales. Is this the sort of pattern you would expect?

**b.** Copy and complete this table to show the relationship between price per shirt and the expected total value of the shirt sales.

**Projected Shirt Sales**

<table>
<thead>
<tr>
<th>Price per Shirt</th>
<th>$5</th>
<th>$10</th>
<th>$15</th>
<th>$20</th>
<th>$25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Shirt Sales</td>
<td>50</td>
<td>40</td>
<td>30</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Value of Shirt Sales</td>
<td>$250</td>
<td>$400</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**c.** How would you describe the relationship between price per shirt and expected total value of shirt sales? Is this the sort of pattern you would expect?

**d.** Use grid paper to make coordinate graphs of the data like the ones started below.

![Projected Shirt Sales Graph](image1)

**e.** Explain how your answers to parts (a) and (c) are shown in the graphs.

Investigation 2 Analyzing Graphs and Tables
5. A camping-supply store rents camping gear for $25 per person.
   a. Make a table of the total rental charges for 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, and 50 campers.
   b. Make a coordinate graph using the data in your table.
   c. Compare the pattern of change in your table and graph with patterns you found in Exercise 3. Describe the similarities and differences between the two sets of data.

6. The tour operators need to rent a truck to transport camping gear, clothes, and bicycle repair equipment. They check prices at two truck-rental companies.
   a. East Coast Trucks charges $4.25 for each mile driven. Make a table of the charges for 0, 25, 50, 75, 100, 125, 150, 175, 200, 225, 250, 275, and 300 miles.
   b. Philadelphia Truck Rental charges $40 per day and an additional $2.00 for each mile driven. Make a table of the charges for renting a truck for five days and driving it 0, 25, 50, 75, 100, 125, 150, 175, 200, 225, 250, 275, and 300 miles.
   c. On one coordinate grid, plot the charge plans for both rental companies. Use a different color to mark each company’s plan.
   d. Based on your work in parts (a)–(c), which company offers the better deal? Explain.
7. The table shows fees for using a campsite at a state park from 1 day up to the park limit of 10 days.

<table>
<thead>
<tr>
<th>Days of Use</th>
<th>Campsite Fee</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$20</td>
</tr>
<tr>
<td>2</td>
<td>$30</td>
</tr>
<tr>
<td>3</td>
<td>$40</td>
</tr>
<tr>
<td>4</td>
<td>$50</td>
</tr>
<tr>
<td>5</td>
<td>$60</td>
</tr>
<tr>
<td>6</td>
<td>$70</td>
</tr>
<tr>
<td>7</td>
<td>$75</td>
</tr>
<tr>
<td>8</td>
<td>$80</td>
</tr>
<tr>
<td>9</td>
<td>$85</td>
</tr>
<tr>
<td>10</td>
<td>$90</td>
</tr>
</tbody>
</table>

a. Make a coordinate graph using the table.
b. Does it make sense to connect the points on your graph? Why or why not?
c. Describe the pattern relating the variables days of use and campsite fee.

8. Suppose a motion detector tracks the time and the distance traveled as you walk 40 feet in 8 seconds. Match the following (time, distance) graphs with the “stories” that describe each walk.

a. You walk at a steady pace of 5 feet per second.
b. You walk slowly at first and then steadily increase your walking speed.
c. You walk rapidly at first, pause for several seconds, and then walk at an increasing rate for the rest of the trip.
d. You walk at a steady rate for 3 seconds, pause for 2 seconds, and then walk at a steady rate for the rest of the trip.
e. You walk rapidly at first, but gradually slow down as the end of the trip nears.

9. For each walk in Exercise 8, complete a (time, distance) table like the one below. Use numbers that will match the pattern of the walk and its graph.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (feet)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>40</td>
</tr>
</tbody>
</table>
10. The graphs below show five patterns of change in the price per gallon of gasoline. Match each \((time, price)\) graph with the “story” it tells.

- **a.** The price declined at a steady rate.
- **b.** The price did not change.
- **c.** The price rose rapidly, then leveled off for a while, and then declined rapidly.
- **d.** The price rose at a steady rate.
- **e.** The price dropped rapidly at first and then at a slower rate.

11. **Multiple Choice** Jamie is going to Washington, D.C., to march in a parade with his school band. He plans to set aside $25 at the end of each month to use for the trip. Choose the graph that shows how Jamie’s savings will build as time passes.

- **A.**
- **B.**
- **C.**
- **D.** None of these is correct.
12. The graph shows how the temperature changed during an all-day hike by students.

<table>
<thead>
<tr>
<th>Elapsed Time (hr)</th>
<th>Temperature During Hike (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. What was the maximum temperature and when did it occur?
b. When was the temperature rising most rapidly?
c. When was the temperature falling most rapidly?
d. When was the temperature about 24°C?
e. The hikers encounter a thunderstorm with rain. When do you think this happened?
Jacy works at a department store. This graph shows parking costs at the parking garage Jacy uses.

13. **Multiple Choice** How much does Jacy spend to park for less than a half hour?
   - F. $0.50
   - G. $0.75
   - H. $1
   - J. $1.50

14. **Multiple Choice** How much does Jacy spend to park for 4 hours and 15 minutes?
   - A. $6
   - B. $6.50
   - C. $6.75
   - D. $7

**Connections**

15. The area of a rectangle is the product of its length and its width.
   a. Find all whole number pairs of length and width values that give an area of 24 square meters. Record the pairs in a table.

   **Rectangles with an Area of 24 m²**
   
<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>□ □ □ □</td>
<td>□ □ □ □</td>
</tr>
</tbody>
</table>

   b. Make a coordinate graph of the (length, width) data from part (a).
   c. Connect the points on your graph if it makes sense to do so. Explain your decision.
   d. Describe the relationship between length and width for rectangles of area 24 square meters.

16. The perimeter of any rectangle is the sum of its side lengths.
   a. Make a table of all possible whole-number pairs of length and width values for a rectangle with a perimeter of 18 meters.
   b. Make a coordinate graph of the (length, width) data from part (a).
   c. Connect the points on your graph if it makes sense to do so. Explain your decision.
   d. Describe the relationship between length and width for rectangles of perimeter 18 meters, and explain how that relationship is shown in the table and graph.
17. The table below shows the winners and the winning times for the women’s Olympic 400-meter dash since 1964.

<table>
<thead>
<tr>
<th>Year</th>
<th>Name</th>
<th>Time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1964</td>
<td>Celia Cuthbert, AUS</td>
<td>52.0</td>
</tr>
<tr>
<td>1968</td>
<td>Colette Besson, FRA</td>
<td>52.0</td>
</tr>
<tr>
<td>1972</td>
<td>Monika Zehrt, E. GER</td>
<td>51.08</td>
</tr>
<tr>
<td>1976</td>
<td>Irena Szewinska, POL</td>
<td>49.29</td>
</tr>
<tr>
<td>1980</td>
<td>Martia Koch, E. GER</td>
<td>48.88</td>
</tr>
<tr>
<td>1984</td>
<td>Valerie Brisco-Hooks, USA</td>
<td>48.83</td>
</tr>
<tr>
<td>1988</td>
<td>Olga Bryzgina, USSR</td>
<td>48.65</td>
</tr>
<tr>
<td>1992</td>
<td>Marie-Jose Perec, FRA</td>
<td>48.83</td>
</tr>
<tr>
<td>1996</td>
<td>Marie-Jose Perec, FRA</td>
<td>48.25</td>
</tr>
<tr>
<td>2000</td>
<td>Cathy Freeman, AUS</td>
<td>49.11</td>
</tr>
<tr>
<td>2004</td>
<td>Tonique Williams-Darling, BAH</td>
<td>49.41</td>
</tr>
</tbody>
</table>

a. Make a coordinate graph of the \((\text{year}, \text{time})\) information. Choose a scale that allows you to see the differences between the winning times.

b. What patterns do you see in the table and graph? Do the winning times seem to be rising or falling? In which year was the best time earned?

18. The circumference of a circle is related to its radius by the formula \(C = 2 \times \pi \times r\).

The area of a circle is related to its radius by the formula \(A = \pi \times r^2\).

a. Make a table showing how the circumference of a circle increases as the radius increases in 1-unit steps from 1 to 6. Make sure to express the circumferences in terms of \(\pi\). Then describe the pattern relating those two variables.

b. Make a table showing how the area of a circle increases as the radius increases in 1-unit steps from 1 to 6. Make sure to express the areas in terms of \(\pi\). Then describe the pattern relating those two variables.
19. Here are the box-office earnings for a movie during each of the first eight weeks following its release.

**Box Office Earnings**

<table>
<thead>
<tr>
<th>Weeks in Theaters</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly Earnings (millions)</td>
<td>$16</td>
<td>$22</td>
<td>$18</td>
<td>$12</td>
<td>$7</td>
<td>$4</td>
<td>$3</td>
<td>$1</td>
</tr>
</tbody>
</table>

a. Make a coordinate graph showing the weekly earnings after each week. Because a film’s weekly earnings depend on the number of weeks it is in theaters, put the weeks in theaters on the x-axis and the weekly earnings on the y-axis.

b. Explain how the weekly earnings changed as time passed. How is this pattern of change shown in the table and the graph? Why might this change have occurred?

c. What were the total earnings of the movie in the eight weeks?

d. Make a coordinate graph showing the total earnings after each week.

e. Explain how the movie’s total earnings changed over time. How is this pattern of change shown in the table and the graph? Why might this change have occurred?

**Extensions**

20. Use what you know about decimals to find coordinates of five points that lie on the line segment between the labeled points on each graph:

a. [Diagram of graph with points (0, 0) and (2, 2)]

b. [Diagram of graph with points (0, 0) and (0.2, 0.2)]
21. The graphs below each show relationships between independent (x-axis) and dependent (y-axis) variables. However, the scales on the coordinate axes are not the same for all the graphs.

**Graph 1**

**Graph 2**

**Graph 3**

**Graph 4**

a. Which graph shows the dependent variable increasing most rapidly as the independent variable increases?

b. Which graph shows the dependent variable increasing most slowly as the independent variable increases?
22. To raise money, students plan to hold a car wash. They ask some adults how much they would pay for a car wash. The table below shows the results of their research.

<table>
<thead>
<tr>
<th>Car Wash Price</th>
<th>$4</th>
<th>$6</th>
<th>$8</th>
<th>$10</th>
<th>$12</th>
<th>$14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Customers</td>
<td>120</td>
<td>105</td>
<td>90</td>
<td>75</td>
<td>60</td>
<td>45</td>
</tr>
</tbody>
</table>

a. Make a coordinate graph of the \((price, customers)\) data. Connect the points if it makes sense to do so.

b. Describe the pattern relating the price to the number of customers. Explain how the table and the graph show the pattern.

c. Based on the pattern, what number of customers would you predict if the price were $16? What number would you predict if the price were $20? What if the price were $2?

23. a. Copy and complete the table below, using the information from Exercise 22.

<table>
<thead>
<tr>
<th>Car Wash Price</th>
<th>$4</th>
<th>$6</th>
<th>$8</th>
<th>$10</th>
<th>$12</th>
<th>$14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Customers</td>
<td>120</td>
<td>105</td>
<td>90</td>
<td>75</td>
<td>60</td>
<td>45</td>
</tr>
<tr>
<td>Projected Income</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Make a graph of the \((price, projected income)\) data. Connect the points if it makes sense to do so.
c. Describe the pattern relating the price and the projected income. Explain how the table and the graph show the pattern. Explain why the pattern does or does not make business sense to you.

d. Suppose the shopping center where the students plan to hold the car wash will charge the students $1.50 per car for water and cleaning supplies. How can you use this factor to find the profit from the car wash for various prices?

24. Adriana is at a skateboard park that has tracks shaped like regular polygons. Recall that a regular polygon is a polygon with congruent sides and congruent angles. Here are some examples:

At each vertex of a track, Adriana must make a turn. The size of the turn relates to the number of sides in the polygon. For example, at each vertex of the triangle track, she must make a 120° turn.

a. Copy and complete the table below to show how the size of the turn Adriana must make at each vertex is related to the number of sides of the polygon.

<table>
<thead>
<tr>
<th>Track Turns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Sides</td>
</tr>
<tr>
<td>Degrees in Turn</td>
</tr>
</tbody>
</table>

b. Make a coordinate graph of the (sides, degrees) data.

c. What pattern of change do you see in the degrees Adriana must turn as the number of sides increases? How does the table show that pattern? How does the graph show that pattern?
The problems in this investigation asked you to think about patterns relating the values of variables. These questions will help you to summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. Explain what the word variable means in mathematics.
2. What does it mean to say that two variables are related?
3. a. Suppose the y-values increase as the x-values increase. How is this indicated in a table? How is this indicated in a graph?
   b. Suppose the y-values decrease as the x-values increase. How is this indicated in a table? How is this indicated in a graph?
4. In a coordinate graph of two related variables, when does it make sense to connect the points?
In the last investigation, you used tables and graphs of relationships to find values of one variable for given values of the other variable. In some cases, you could only estimate or predict a value.

For some relationships, you can write an equation, or formula, to show how the variables are related. Using an equation is often the most accurate way to find values of a variable.

In this investigation, you will use the patterns in tables to help you write equations for relationships. You will then use your equations to compute values of the dependent variable for specific values of the independent variable.

On the last day of the Ocean Bike Tour, the riders will be near Wild World Amusement Park. Liz and Malcolm want to plan a stop there. They consider several variables that affect their costs and the time they can spend at Wild World.

- What variables do you think are involved in planning for the amusement-park trip?
- How are those variables related to each other?
Malcolm finds out that it costs $21 per person to visit Wild World. Liz suggests they make a table or graph relating admission price to the number of people. However, Malcolm says there is a simple rule for calculating the cost:

The cost in dollars is equal to 21 times the number of people.

He writes the rule as an equation:

\[ \text{cost} = 21 \times \text{number of people} \]

Liz shortens Malcolm’s equation by using single letters to stand for the variables. She uses \( c \) to stand for the cost and \( n \) to stand for the number of people:

\[ c = 21 \times n \]

When you multiply a number by a letter variable, you can leave out the multiplication sign. So, \( 21n \) means \( 21 \times n \). You can shorten the equation even more:

\[ c = 21n \]

The equation \( c = 21n \) involves one calculation. You multiply the number of customers \( n \) by the cost per customer $21. Many common equations involve one calculation.

### Problem 3.1 Equations With One Operation

The riders visited Wild World and the tour is over. They put their bikes and gear into vans and head back to Atlantic City, 320 miles away. On their way back, they try to calculate how long the drive home will take. They use a table and a graph to estimate their travel time for different average speeds.

**A.** Copy and complete the table.

<table>
<thead>
<tr>
<th>Time (hr)</th>
<th>Distance for Speed of 50 mi/h</th>
<th>Distance for Speed of 55 mi/h</th>
<th>Distance for Speed of 60 mi/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>■</td>
<td>■</td>
</tr>
<tr>
<td>1</td>
<td>50</td>
<td>■</td>
<td>■</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>■</td>
<td>■</td>
</tr>
<tr>
<td>3</td>
<td>■</td>
<td>■</td>
<td>■</td>
</tr>
<tr>
<td>4</td>
<td>■</td>
<td>■</td>
<td>■</td>
</tr>
<tr>
<td>5</td>
<td>■</td>
<td>■</td>
<td>■</td>
</tr>
<tr>
<td>6</td>
<td>■</td>
<td>■</td>
<td>■</td>
</tr>
</tbody>
</table>
B. Copy and complete the graph for all three speeds below. Use a different color for each speed.

![Distance at Different Speeds Graph](image)

C. Do the following for each of the three average speeds:
   1. Look for patterns relating distance and time in the table and graph. Write a rule in words for calculating the distance traveled in any given time.
   2. Write an equation for your rule, using letters to represent the variables.
   3. Describe how the pattern of change shows up in the table, graph, and equation.

D. For each speed, (50, 55, and 60 mph) tell how far you would travel in the given time. Explain how you can find each answer by using the table, the graph, and the equation.
   1. 3 hours
   2. 4 1/2 hours
   3. 5 1/4 hours

E. For each speed, find how much time it will take the students to reach these cities on their route:
   1. Atlantic City, New Jersey, about 320 miles from Norfolk
   2. Baltimore, Maryland, about 3/4 of the way from Norfolk to Atlantic City

ACE Homework starts on page 55.
3.2 Writing More Equations

The equations you wrote in Problem 3.1 involved only multiplication. Some equations involve two or more arithmetic operations (+, −, ×, ÷). To write such equations, you can reason just as you do when you write one-operation equations:

Determine what the variables are.
Work out some specific numeric examples and examine them carefully. What patterns do you see? What is the role of each variable in the calculation?
Write a rule in words to describe the general pattern in the calculations.
Convert your rule to an equation with letter variables and symbols.
Think about whether your equation makes sense. Test it for a few values to see if it works.

Problem 3.2 Equations With Two Operations

When Liz tells Theo about the idea to visit Wild World, he suggests she check to see whether the park offers special prices for large groups. She finds this information on the park’s Web site:

A. 1. Find the price of admission for a group of 20 people, a group of 35 people, and a group of 42 people.

2. Describe in words how you can calculate the admission price for a group with any number of people.
3. Write an equation for the admission price $p$ for a group of $n$ people.

4. Sketch a graph to show the admission price for a group of any size.

5. How does the pattern of change show up in the equation and graph? How is this pattern similar to the pattern in Problem 3.1? How is it different?

B. Admission to Wild World includes a bonus card with 100 points that can be spent on rides. Rides cost 6 points each.

1. Copy and complete the table below to show a customer’s bonus card balance after each ride. Pay close attention to the values in the Number of Rides row.

<table>
<thead>
<tr>
<th>Number of Rides</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
<th>13</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points on Card</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Describe in words how you can calculate the number of points left after any number of rides.

3. Write an equation showing the relation between the number of rides and the points left on the bonus card. Use letters to represent the variables.

4. Sketch a graph of the data.

5. How does the pattern of change between the variables show up in the equation and graph? How is this pattern similar to the pattern in Question A? How is it different?

C. Liz wonders whether they should rent a golf cart to carry the riders’ backpacks at the park. The equation $c = 20 + 5h$ shows the cost $c$ in dollars of renting a cart for $h$ hours:

1. Explain what information the numbers and variables in the equation represent.

2. Use the equation to make a table for the cost of renting a cart for 1, 2, 3, 4, 5, and 6 hours.

3. Make a graph of the data.

4. Describe how the pattern of change between the two variables shows up in the table, graph, and equation.

ACE Homework starts on page 55.
The students think that $350 is a fair price to charge for the tour. Sidney wants to be certain Ocean Bike Tours will make a profit if they charge $350. She starts making the table below.

### Tour Revenue and Expenses

<table>
<thead>
<tr>
<th>Number of Customers</th>
<th>Revenue</th>
<th>Bike Rental</th>
<th>Food and Camp Costs</th>
<th>Total Expenses</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$350</td>
<td>$30</td>
<td>$125</td>
<td>$350</td>
<td>$0</td>
</tr>
<tr>
<td>2</td>
<td>$700</td>
<td>$60</td>
<td>$250</td>
<td>$700</td>
<td>$0</td>
</tr>
<tr>
<td>3</td>
<td>$1,050</td>
<td>$90</td>
<td>$375</td>
<td>$1,050</td>
<td>$0</td>
</tr>
</tbody>
</table>

**Problem 3.3 Equations for Revenue, Expenses, and Profit**

A. Extend and complete Sidney’s table for 1 to 6 customers.

B. Write a rule in words and an equation for calculating the
   1. revenue \( r \) for \( n \) customers
   2. total expenses \( e \) for \( n \) customers
   3. profit \( p \) for \( n \) customers

C. Use the equations you wrote in Question B to find the revenue, expenses, and profit for 20 customers and for 31 customers.

D. Sidney forgot that the tour operators need to rent a van to carry equipment. The rental cost for the van will be $700.
   1. How does this expense affect the equation for total expenses?
   2. How does this expense affect the equation for profit?

ACE Homework starts on page 55.
Applications

1. The El Paso Middle School girls’ basketball team is going from El Paso to San Antonio for the Texas state championship game. The trip will be 560 miles. Their bus travels at an average speed of 60 miles per hour.

   a. Suppose the bus travels at an almost steady speed throughout the trip. Make a table and a graph of time and distance data for the bus.

   b. Estimate the distance the bus travels in 2 hours, $2\frac{3}{4}$ hours, $3\frac{1}{2}$ hours, and 7.25 hours.

   c. How are 2 hours and the distance traveled in 2 hours represented in the table? How are they shown on the graph?

   d. How are $2\frac{3}{4}$ hours and the distance traveled in $2\frac{3}{4}$ hours represented in the table? How are they shown on the graph?

   e. Describe in words a rule you can use to calculate the distance traveled for any given time on this trip.

   f. The bus route passes through Sierra Blanca, which is 90 miles from El Paso. About how long does it take the bus to get to Sierra Blanca?

   g. The bus route also passes through Balmorhea, which is $\frac{1}{3}$ of the way from El Paso to San Antonio. About how long does it take the bus to get to Balmorhea?

   h. How long does it take the bus to complete its 560-mile trip to San Antonio?
2. Celia writes the equation \( d = 8t \) to represent the distance in miles \( d \) that bikers could travel in \( t \) hours at a speed of 8 miles per hour.
   
a. Make a table that shows the distance traveled every half hour, up to 5 hours, if bikers ride at this constant speed.
   
b. How far would bikers travel in 1 hour, 6 hours, 8.5 hours, and 10 hours?

3. The equation \( d = 70t \) represents the distance in miles covered after traveling at 70 miles per hour for \( t \) hours.
   
a. Make a table that shows the distance traveled every half hour from 0 hours to 4 hours.
   
b. Sketch a coordinate graph that shows the distance traveled between 0 and 4 hours.
   
c. What is \( d \) when \( t = 2.5 \) hours?
   
d. What is \( t \) when \( d = 210 \) miles?
   
e. You probably made your graph by plotting points. In this situation, would it make sense to connect these points?

4. a. Use the table to write an equation that relates lunch cost \( L \) and number of riders \( n \).

   **Bike Tour Box Lunch Costs**
   
<table>
<thead>
<tr>
<th>Riders</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lunch Cost</td>
<td>$4.25</td>
<td>$8.50</td>
<td>$12.75</td>
<td>$17.00</td>
<td>$21.25</td>
<td>$25.50</td>
<td>$29.75</td>
<td>$34.00</td>
<td>$38.25</td>
</tr>
</tbody>
</table>

   b. Use your equation to find the lunch cost for 25 riders.
   
c. How many riders could eat lunch for $89.25?

   For Exercises 5–7, use the equation to complete the table.

5. \( y = 4x + 3 \)
   
   \[
   \begin{array}{c|c|c|c|c|c}
   x & 1 & 2 & 5 & 10 & 20 \\
   \hline
   y & & & & & \\
   \end{array}
   \]

6. \( m = 100 - k \)
   
   \[
   \begin{array}{c|c|c|c|c|c}
   k & 1 & 2 & 5 & 10 & 20 \\
   \hline
   m & & & & & \\
   \end{array}
   \]

7. \( d = 3.5t \)
   
   \[
   \begin{array}{c|c|c|c|c|c}
   t & 1 & 2 & 5 & 10 & 20 \\
   \hline
   d & & & & & \\
   \end{array}
   \]
8. Sean is buying a new DVD player and speakers for $315. The store offers him an interest-free payment plan that allows him to pay in monthly installments of $25.
   a. How much will Sean still owe after one payment? After two payments? After three payments?
   b. Use $n$ to stand for the number of payments and $a$ for the amount still owed. Write an equation for calculating $a$ for any value of $n$.
   c. Use your equation to make a table and a graph showing the relationship between $n$ and $a$.
   d. As $n$ increases by 1, how does $a$ change? How is this change shown in the table? How is it shown on the graph?
   e. How many payments will Sean have to make in all? How is this shown in the table? How is this shown on the graph?

For Exercises 9–12, express each rule as an equation. Use single letters to stand for the variables. Identify what each letter represents.

9. The area of a rectangle is its length multiplied by its width.

10. The number of hot dogs needed for the picnic is two for each student.

11. The amount of material needed to make the curtains is 4 square yards per window.

12. Taxi fare is $2.00 plus $1.10 per mile.

13. The sales tax in a state is 8%. Write an equation for the amount of tax $t$ on an item that costs $p$ dollars.

14. An airplane is traveling at 550 miles per hour. Write an equation for the distance $d$ the plane travels in $h$ hours.

15. Potatoes sell for $0.25 per pound at the produce market. Write an equation for the cost $c$ of $p$ pounds of potatoes.

16. A cellular family phone plan costs $49 per month plus $0.05 per minute of long-distance service. Write an equation for the monthly bill $b$ when $m$ minutes of long-distance service are used.
For Exercises 17–19, describe the relationship between the variables in words and with an equation.

17. 

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>8</td>
<td>20</td>
<td>40</td>
<td>80</td>
</tr>
</tbody>
</table>

18. 

<table>
<thead>
<tr>
<th>s</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>t</td>
<td>49</td>
<td>48</td>
<td>47</td>
<td>44</td>
<td>38</td>
</tr>
</tbody>
</table>

19. 

<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>z</td>
<td>6</td>
<td>11</td>
<td>16</td>
<td>21</td>
<td>26</td>
</tr>
</tbody>
</table>

20. **Multiple Choice** Which equation describes the relationship in the table?

<table>
<thead>
<tr>
<th>n</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
</tr>
</tbody>
</table>

A. \( C = 10n \)  
B. \( C = 10 + n \)  
C. \( C = 10 \)  
D. \( C = 10 + 10n \)  

**Connections**

21. The perimeter \( P \) of a square is related to the side length \( s \) by the formula \( P = 4s \). The area, \( A \), is related to the side length by the formula \( A = s \times s \), or \( A = s^2 \).

a. Make a table showing how the perimeter of a square increases as the side length increases from 1 to 6 in 1-unit steps. Describe the pattern of change.

b. Make a table showing how the area of a square increases as the side length increases from 1 to 6. Describe the pattern of change.

For Exercises 22–27, find the indicated value or values.

22. the mean, or average, of 4.5 and 7.3

23. the area of a circle with radius 6 centimeters

24. the sum of the angle measures in a triangle, in a parallelogram, in a pentagon, and in a hexagon
25. the 10th odd number (1 is the first odd number, 3 is the second odd number, and so on.)

26. the area of a triangle with a base of 10 centimeters and a height of 15 centimeters

27. $3^3 \times 5^2 \times 7$

28. The wheels on Kai’s bike are 27 inches in diameter. His little sister, Masako, has a bike with wheels that are 20 inches in diameter. Kai and Masako are on a bike ride.
   a. How far does Kai go in one complete turn of his wheels?
   b. How far does Masako go in one complete turn of her wheels?
   c. How far does Kai go in 500 turns of his wheels?
   d. How far does Masako go in 500 turns of her wheels?
   e. How many times do Kai’s wheels have to turn to cover 100 feet?
   f. How many times do Masako’s wheels have to turn to cover 100 feet? To cover 1 mile?

29. Bicycles that were popular in the 1890s were called “penny farthing” bicycles. These bikes had front wheels with diameters as great as 5 feet! Suppose the front wheel of these bicycles have a diameter of 5 feet.

   a. What is the radius of the front wheel?
   b. How far will one bike travel in 100 turns of the front wheel?
   c. How many times will the front wheel turn in a 3-mile trip?
   d. Compare the number of times the wheels of Masako’s bike turn in a 1-mile trip [see part (f) of Exercise 28] with the number of times the front wheel of this penny-farthing bike turns in a 3-mile trip. Why are the numbers related this way?
Write a formula for the given quantity.

30. the area $A$ of a rectangle with length $\ell$ and width $w$

31. the area $A$ of a parallelogram with base $b$ and height $h$

32. the perimeter $P$ of a rectangle with base $b$ and height $h$

33. the mean $m$ of two numbers $p$ and $q$

34. the area $A$ of a circle with radius $r$

35. the sum $S$ of the measures of angles in a polygon of $n$ sides

36. the $n$th odd number, $O$ (1 is the first odd number, 3 is the second odd number, and so on.)

37. the area $A$ of a triangle with base $b$ and height $h$

Complete the table of values for the given equation.

38. $y = x + \frac{1}{2}$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\frac{1}{5}$</th>
<th>$\frac{1}{4}$</th>
<th>$\frac{1}{3}$</th>
<th>$\frac{2}{5}$</th>
<th>$\frac{1}{2}$</th>
<th>$\frac{2}{3}$</th>
<th>$\frac{3}{4}$</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

39. $y = \left(\frac{1}{2}\right)x$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\frac{1}{5}$</th>
<th>$\frac{1}{4}$</th>
<th>$\frac{1}{3}$</th>
<th>$\frac{2}{5}$</th>
<th>$\frac{1}{2}$</th>
<th>$\frac{2}{3}$</th>
<th>$\frac{3}{4}$</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Describe the relationship between $x$ and $y$ in words.

40. 

41. 

42. 

Variables and Patterns
Extensions

43. a. You can calculate the average speed of a car trip if you know the distance and time traveled. Copy and complete the table below.

<table>
<thead>
<tr>
<th>Distance (mi)</th>
<th>Time (hr)</th>
<th>Average Speed (mi/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>145</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>165</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>5.25</td>
<td></td>
</tr>
<tr>
<td>446</td>
<td>6.75</td>
<td></td>
</tr>
<tr>
<td>528</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>862</td>
<td>9.5</td>
<td></td>
</tr>
<tr>
<td>723</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

b. Write a formula for calculating the average speed $s$ for any given distance $d$ and time $t$.

For Exercises 44–47, solve each problem by estimating and checking.

44. The equation $p = 50 + 10n$ gives the admission price $p$ to Wild World for a group of $n$ people. A club’s budget has $500 set aside for a visit to the park. How many club members can go?

45. The equation $b = 100 - 6r$ gives the number of bonus points $b$ left on a Wild World bonus card after $r$ rides.
   a. Rosi has 34 points left. How many rides has she been on?
   b. Dwight has 16 points left. How many rides has he been on?

46. The equation $d = 2.5t$ describes the distance in meters $d$ covered by a canoe-racing team in $t$ seconds. How long does it take the team to go 125 meters? How long does it take them to go 400 meters?

47. The equation $d = 400 - 2.5t$ describes the distance in meters $d$ of a canoe-racing team from the finish line $t$ seconds after a race starts. When is the team 175 meters from the finish line? When is it 100 meters from the finish line?
48. Armen builds models from rods. When he builds bridges, he makes the sides using patterns of triangles like the ones below. The total number of rods depends on the number of rods along the bottom.

Rods along bottom = 3
Total number of rods = 11

Rods along bottom = 4
Total number of rods = 15

a. Copy and complete the table.

<table>
<thead>
<tr>
<th>Rod Bridges</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rods Along the Bottom</strong></td>
</tr>
<tr>
<td><strong>Total Number of Rods</strong></td>
</tr>
</tbody>
</table>

b. Write an equation relating the total number of rods $t$ to the number of rods along the bottom $b$. Explain how the formula you write relates to the way Armen puts the rods together.

c. What do you know about the properties of triangles and rectangles that makes the design above better than the one below?

49. The students in Problem 3.3 decide to visit Wild World Amusement Park on the tour. They include the cost of this and the van in their revenue and expenses. How does this affect the equation for profit?
In this investigation, you wrote equations to express relationships between variables. The following questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. What decisions do you need to make when you write an equation to represent a relationship between variables?

2. In what ways are equations useful?

3. In this unit, you have represented relationships with tables, graphs, and equations. List some advantages and disadvantages of each of these representations.
In the last investigation, you wrote equations to describe patterns and to show how variables are related. Such equations are used in mathematics, science, economics, and many other subject areas. Tables, graphs, and equations are all useful ways of representing relationships between variables. When you have an equation relating variables, you can use a graphing calculator to make a graph or table of the relationship quickly.

**Making and Using Calculator Tables**

Suppose you want to use your calculator to make a table of values for the formulas for the circumference $C$ and area $A$ of a circle with radius $r$:

$$C = 2\pi r \quad \text{and} \quad A = \pi r^2.$$  

To enter the equations into your calculator, press $\text{Y=}$ to get a screen like the one below.

![Calculator screen](image)
On most calculators, you enter the independent variable as X; the
dependent variable is Y. Enter the equation for circumference as Y1 and
the equation for area as Y2 as shown below.

Next, press \[2\text{nd}\] TblSet and enter the table settings shown below. The
settings indicate that the x-values (radius values) in the table should start at
0 and increase in steps of 1.

Press \[2\text{nd}\] TABLE, and you will see the table.

You can use the scroll keys to see more values for radius, circumference,
and area.

Getting Ready for Problem 4.1

- What does the number 28.274 in the third column mean?
- What does 25.133 in the second column mean?
- What does 5 in the first column mean?
In the following problem, you will use calculator tables to explore relationships between variables. You will scan the tables to solve problems or look for interesting patterns in the entries.

**Problem (4.1) Making and Using Calculator Tables**

**A.** The equation \( p = 50 + 10n \) represents the Wild World admission price in dollars \( p \) for a group of \( n \) people. To study this relationship, enter the equation \( Y1 = 50 + 10X \) into your calculator.

1. Make a calculator table showing \((n, p)\) values for \( n = 1, 2, 3, \) and so on.
2. Scan the table to find the admission price for a group of 26 people.
3. Find the value of \( n \) for which \( p = 480 \). Explain what this entry tells you about the admission price and the number of people.
4. Use the table to find the value of \( n \) for which \( 950 = 50 + 10n \). Explain what this table entry tells you about the admission price and the number of people.

**B.** The equation \( d = 2.5t \) represents the distance a canoe team paddles in meters \( d \) in \( t \) seconds.

1. Make a calculator table showing \((t, d)\) values for \( t = 4, 8, 12, \) and so on.
2. Use the table to find the distance the team paddles in 40 seconds.
3. Use the table to find the value of \( t \) for which \( 2.5t = 437.5 \). Explain what this entry tells you about the time and distance traveled.

**C.** The equation \( b = 100 - 6r \) gives the number of bonus points \( b \) left on a Wild World bonus card after \( r \) rides.

1. Make a calculator table showing \((r, b)\) values for \( r = 0, 2, 4, \) and so on.
2. How does the number of bonus points change as the number of rides increases in steps of 2? Why does that pattern occur?
3. Use your table to find a value of \( r \) for which \( 100 - 6r = 10 \). (Adjust the table settings if you need to.) Explain what this entry tells you about the number of rides and the bonus points left on the card.

**ACE** Homework starts on page 72.
4.2 Making and Using Calculator Graphs

Graphing calculators can help you make and study graphs. Just as when you make a graph with pencil and paper, the key step is choosing ranges and scales for the axes.

Suppose you want to graph \( y = 1.5x + 2 \). Press \( \text{Y}= \) and enter the equation, just as you do when you make a table. Next, set the boundaries of the graphing window. Press \( \text{WINDOW} \) to see the screen below. The settings shown here are the *standard* window settings.

Press \( \text{GRAPH} \) to see what the graph looks like in this window.

Suppose you want to focus on the part of the graph that shows positive \( x \)- and \( y \)-values. Press \( \text{WINDOW} \) again and change the settings. Use the settings shown below at the left. Then press \( \text{GRAPH} \) to see the screen on the right.
To find coordinates of particular points on the graph, press \( \text{TRACE} \). Use the arrow keys to move the cursor along the graph and see coordinates of highlighted points.

When the cursor is on the line, the coordinates appear at the bottom of the screen and the equation appears in the upper left.

**Getting Ready for Problem 4.2**

Experiment with your graphing calculator and the following equations.

Graph one set of equations at a time. Use the standard window.

- **Set 1**
  - \( y = 3x - 4 \)
  - \( y = x^2 \)
  - \( y = 3x + 2 \)

- **Set 2**
  - \( y = 5 \)
  - \( y = 3x \)
  - \( y = 1x \)

- **Set 3**
  - \( y = 2x + 3 \)
  - \( y = 2x - 5 \)
  - \( y = (0.5)x + 2 \)

- **Set 4**
  - \( y = 2x \)
  - \( y = 2 \times x \)
  - \( y = 2^x \)

Answer the following questions for each set of equations.

- Which, if any, of the graphs show similar patterns of change? How are the graphs similar? How are the equations for the graphs similar?
- How are the graphs different? How are the equations for the graphs different?

**Problem 4.2 Making and Using Calculator Graphs**

A. One popular attraction at Wild World is the climbing wall. Supports for the wall are frames of squares made from steel rods. The number of rods depends on the number of square sections in the frame.

- 1 section: 4 rods
- 2 sections: 7 rods
- 3 sections: 10 rods
1. An engineer at Wild World wrote the equation \( r = 3s + 1 \) to relate the number of sections and the number of rods. Do you think this formula is correct? Explain. What does each variable in the equation represent?

2. Use your calculator to graph the engineer’s equation. Use the window settings \( X_{\text{min}} = 0, X_{\text{max}} = 15, Y_{\text{min}} = 0, \) and \( Y_{\text{max}} = 50. \) Sketch your graph on axes like these. Give your graph a title.

3. Press \( \text{TRACE} \) and use the arrow keys to move along the graph. Locate a point with a \( y \)-value of about 19. Mark this point on your sketch and label it with its coordinates. Then, find a point with a \( y \)-value of about 43. Mark and label this point on your sketch.

4. Explain what the coordinates of each point in part (3) tell you about the number of sections and the number of rods.

5. Use your calculator graph to find a value of \( s \) for which \( 3s + 1 = 28. \) Give the coordinates of that point, and explain what they tell you about sections and rods.

B. Make a calculator graph of \( b = 100 - 6r \), which gives the bonus card balance \( b \) after \( r \) rides. Experiment with the window settings until you have a good view of the graph. (You might find it helpful to look at the table you made in Problem 4.1.)

1. Make a sketch of the graph. Label the axes with variable names and scales. Add a descriptive title to your graph.

2. Find a point on the calculator graph for which \( b = 58. \) (The symbol \( \approx \) means “is approximately equal to.”) Mark this point on your sketch and label it with its coordinates. Explain what the coordinates tell you about the number of rides taken and the balance on the card.

3. Use your calculator graph to find a value of \( r \) for which \( 100 - 6r = 22. \) Give the coordinates of that point and explain what they tell you about the rides taken and the card balance.

**ACE** Homework starts on page 72.
The bike tour was a success! So many people signed up for the tour that the students decide to offer an additional two-day bike tour. Two bike rental companies submit bids to the students. Their price quotes are shown here.

In the next problem, you will use your graphing calculator to analyze the price quotes and advise Ocean Bike Tours about which company to use.
Comparing Relationships

Study the price quotes from the two companies. The following questions will help you analyze the information.

A. Make a table showing the costs for renting bikes from each company. Include costs for 5, 10, 15, 20, 25, 30, 35, and 40 bikes in your table.

B. Plot the (number of bikes, rental cost) data for both companies on a graph like the one below. Use a different color or plotting symbol for each company.

C. For each company, write an equation relating the number of bikes rented to the total cost. Tell what the letter variables in your equation represent.

D. Enter your equations into your calculator as Y1 and Y2. Graph both equations in the same window. Compare the calculator graph with the graph you made by hand.

E. Is there a number of bikes for which both companies charge the same rental fee? How can the graph help you find the answer?

F. How does the graph show the number of bikes for which Cycle the Coast is the most economical? How does it show the number of bikes for which Speedy Wheels offers the better deal?

G. What advice would you give Ocean Bike Tours about which company to rent bikes from? Use your analysis of the two plans to justify your advice.

Homework starts on page 72.
Applications

1. Sean bought a DVD player and a receiver. The store offered him an interest-free payment plan with weekly installments. Sean figured out that after \( n \) weeks of payments, he would still owe \( 175 - 7.5n \) dollars.
   
a. Make a calculator table showing what Sean owes after payments 1, 2, 3, and so on. Study the table to figure out the amount of his weekly payment. How is this amount shown in the table pattern?
   
b. Scan the table to find out how much Sean will owe after 20 weeks. Record the \((x, y)\) table entry that shows the answer.
   
c. When will the amount Sean owes fall below $100? Which \((x, y)\) table entry shows the answer to this question?
   
d. When will Sean have paid for his items in full? How is this shown in the table?

2. Trevor entered an equation into his graphing calculator, and the calculator displayed this graph and table. The graph is shown in the standard window.
   
a. What is the value of \( y \) when \( x = 6 \)? How is this shown in the table? How is it shown in the graph?
   
b. What equation did Trevor enter into his calculator?

3. Ziamara used her calculator to make a graph of \( y = 3x \). She noticed that the point \((0, 0)\) was on the graph. Name three other points on the graph. Explain how you found these points.
4. The operators of Ocean Bike Tours consider leasing a small bus. They compare two companies. Superior Buses charges $5 for each mile driven. East Coast Transport charges $1,000, plus $2.50 per mile driven.

   a. For each company, write an equation to show how the lease cost \( c \) depends on number of miles driven \( m \).

   b. Enter both equations into your calculator. Choose window settings that make sense for this situation and that show a good view of both graphs. Sketch the graphs, and tell what axes limits (\( X_{\text{min}}, X_{\text{max}}, Y_{\text{min}}, Y_{\text{max}} \)) you used.

   c. Press \( \text{TRACE} \). Trace your graph to estimate the coordinates of the point at which the lease cost is the same for both companies. Explain what the coordinates of the point tell you about the bus-rental situation.

   d. For what driving mileage would the East Coast lease be a better deal? For what mileage would the Superior Buses lease be better? Explain how your answers are shown on the graph from part (b).

5. The Mudville Manatees won the league baseball championship. The manager of the souvenir shop wants to order special shirts and caps to sell to fans. She does market research and predicts these relationships between price in dollars \( p \) and number sold \( n \):

   - Shirts: \( n = 5,000 - 150p \)
   - Caps: \( n = 3,000 - 100p \)

Use these equations to answer the questions below. Making calculator tables and graphs might help.

   a. What are the projected shirt sales if the price is $20 per shirt?

   b. Suppose the manager wants to sell 3,500 shirts. How much should she charge for each shirt?

   c. What are the projected cap sales if the price is $17 per cap?

   d. Suppose the manager wants to sell 1,800 caps. How much should she charge for each cap?
6. The principal of Lincoln Middle School wants to send her top science students on a field trip to the state science center. The trip costs $250 for a bus and driver, plus $17.50 per student for food and admission.

   a. What equation relates the trip cost \( c \) to the number of students who go on the trip \( n \)?
   
   b. What is the cost of sending 30 students? What is the cost for 60 students?
   
   c. How many students can go if the budget allows a maximum cost of $1,000?

Connections

For Exercises 7–10, use the pattern to find the missing entries. Then, write an equation relating the two variables.

7. \[
\begin{array}{c|ccccc}
\text{a} & 0 & 1 & 2 & 3 & 8 & 20 & 100 \\
\text{b} & 0 & 7 & 14 & 21 & 28 & & & \\
\end{array}
\]

8. \[
\begin{array}{c|ccccccc}
\text{x} & 0 & 1 & 2 & 3 & 4 & 8 & 20 & 100 \\
\text{y} & 6 & 7 & 8 & 9 & & & & \\
\end{array}
\]

9. \[
\begin{array}{c|ccccccc}
\text{m} & 0 & 1 & 2 & 3 & 4 & 8 & 20 & 100 \\
\text{n} & 1 & 3 & 5 & 7 & & & & \\
\end{array}
\]

10. \[
\begin{array}{c|ccccccc}
\text{r} & 0 & 1 & 2 & 3 & 4 & 6 & 10 & 20 \\
\text{s} & 0 & 1 & 4 & 9 & 16 & & & \\
\end{array}
\]

11. José used his graphing calculator to find the whole-number factors of 960. Here are the steps he followed:

   \begin{itemize}
   \item \textbf{Step 1:} Enter the equation \( y = 960 \div x \).
   \item \textbf{Step 2:} Set TblStart = 1 and \( \Delta \text{Tbl} = 1 \).
   \item \textbf{Step 3:} Scan the table, looking for whole numbers in the \( y \) column.
   \end{itemize}

   a. Use José’s strategy. Make a list of the factor pairs for 960.
   
   b. Explain why José’s strategy works for 960. How could you modify his strategy to find factors of a different whole number?
   
   c. What is the greatest \( x \)-value you need to check to guarantee you have found all the factors of 960? Explain.
12. Most states add sales tax to the cost of non-food items. Let \( p \) stand for the list price of an item, \( t \) for the additional amount you must pay due to the sales tax, and \( c \) for the total cost of buying the item.

a. What equation relates \( c, p, \) and \( t \)?

b. Suppose a state has a sales tax of 8%. What equation relates \( t \) and \( p \)? What equation relates \( c \) and \( p \)?

c. Enter the equations from part (b) in your calculator. Make a calculator table showing list price, tax, and total cost for items priced from $0 to $100 in steps of $5.

d. What is the total cost of a non-food item that is priced $65?

13. You have seen that many of the costs for the Ocean Bike Tour depend on the number of customers. This table shows a new relationship between the number of customers and the cost of a ferry ride.

<table>
<thead>
<tr>
<th>Customers</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ferry Cost</td>
<td>$2.50</td>
<td>$5.00</td>
<td>$7.50</td>
<td>$10.00</td>
<td>$12.50</td>
<td>$15.00</td>
<td>$17.50</td>
<td>$20.00</td>
<td>$22.50</td>
</tr>
</tbody>
</table>

a. Write an equation for the rule relating ferry cost \( f \) and number of customers \( n \).

b. Use your equation to find the cost if 35 people are on the tour.

c. How many people can go on the ferry if the tour leader has $75?

14. Look back at Exercise 12 in Investigation 1. The first graph shown is the relationship between Amanda’s hunger and the time of day. Could you represent this relationship in a table? Could you represent this relationship with an equation? Explain.
15. You know from your work with polygons that one way to find the sum of the interior angles of a polygon is to divide the shape into triangles by drawing diagonals from one of the vertices, as shown below.

<table>
<thead>
<tr>
<th>3 sides</th>
<th>4 sides</th>
<th>5 sides</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 diagonals</td>
<td>1 diagonal</td>
<td>2 diagonals</td>
</tr>
</tbody>
</table>

a. Copy and complete the table below. The last row should contain formulas for $D$, $S$, and $A$ for a regular polygon with $n$ sides. (Remember that a regular polygon is a polygon in which all sides are the same length and all angles are the same size.)

<table>
<thead>
<tr>
<th>Number of Sides, $n$</th>
<th>Number of Diagonals, $D$ (from a single vertex)</th>
<th>Sum of Interior Angles, $S$</th>
<th>Size of Each Angle if Polygon is Regular, $A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0</td>
<td>180°</td>
<td>60°</td>
</tr>
<tr>
<td>4</td>
<td>■</td>
<td>■</td>
<td>■</td>
</tr>
<tr>
<td>5</td>
<td>■</td>
<td>■</td>
<td>■</td>
</tr>
<tr>
<td>6</td>
<td>■</td>
<td>■</td>
<td>■</td>
</tr>
<tr>
<td>$n$</td>
<td>$D = \cdot$</td>
<td>$S = \cdot$</td>
<td>$A = \cdot$</td>
</tr>
</tbody>
</table>

b. Enter your formula for $D$ into your calculator. Use a table or graph to find the number of sides a polygon must have if you can draw exactly 10 diagonals from a single vertex.

c. Enter your formula for $S$ into your calculator. Use a table or graph to find the sum of the angles for a polygon with 10 sides.

d. Enter your formula for $A$ into your calculator. Using a table or graph, look at the $A$ values for polygons with up to 20 sides. Identify (by giving the number of sides) all regular polygons that have whole number interior angle measures.
16. The area, $A$, of a circle is related to the radius, $r$, by the equation $A = \pi r^2$. Enter this formula into your calculator. Use a calculator graph or table to estimate the answers to the questions below. Make your estimates correct to the nearest tenth.

a. What is the radius of a circle that has area 144 square inches (1 square foot)?

b. What is the radius of a circle that has area 9 square feet (1 square yard)?

Extensions

17. In this unit, you have studied a variety of relationships between variables. The equations below are simple cases of three important types of patterns of change.

\[ y_1 = 2x \quad y_2 = x^2 \quad y_3 = \frac{3}{x} \]

a. For each equation, make a table of $(x, y)$ values for $x = 0, 1, 2, 3,$ and so on. Describe the pattern of change in each table by completing this sentence:

As the value of $x$ increases, the value of $y$ ________________.

Be as precise as you can in describing the patterns. Then, compare each individual pattern of change with the others.

b. Graph the three equations in the same viewing window, with $X_{\text{min}} = 0, X_{\text{max}} = 5, Y_{\text{min}} = 0,$ and $Y_{\text{max}} = 15$. Describe the patterns of change shown by the graphs. Explain how each pattern can be predicted by thinking about the calculations required to find $y$ from $x$. 
18. You can use your graphing calculator to simulate probability experiments that involve rolling dice or flipping coins.

a. Press $Y=$.

Then, press $\text{MATH}$, highlight PRB on the top of the screen, and press 5 to select “randInt(,”

You will be back to the $Y=$ screen. Complete the equation by typing $0 \, \text{INT} \, 1$.

The equation $y = \text{randInt}(0, 1)$ randomly gives a $y$-value of either 0 or 1, no matter what the $x$-value is. Consider 0 to mean “heads” and 1 to mean “tails.”

Press $\text{2nd} \, \text{TblSet}$ and specify $\text{TblStart} = 1$ and $\Delta \text{Tb} = 1$. Then press $\text{2nd} \, \text{TABLE}$. The $X$ column counts the coin tosses, and the $Y1$ column tells you whether the result is heads (0) or tails (1).

Copy and complete the table below.

<table>
<thead>
<tr>
<th>Number of Tosses</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Heads</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of Heads</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of Tails</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Describe patterns in the results. Are the results what you would expect? Explain.
b. Revise the procedures of part (a) to simulate rolling a fair die 50 times. Calculate the number of times and fraction of times the outcome 1 occurs in the 50 rolls. Are the results what you would expect? Explain. (Hint: The equation $y = \text{randInt}(3, 5)$ randomly gives a $y$-value of 3, 4, or 5. The equation $y = \text{randInt}(13, 17)$ randomly gives a $y$-value of 13, 14, 15, 16, or 17. What equation would give random dice rolls?)

19. The bike tour holds a 30-mile race on the last day. They give the two youngest riders, Tony and Sarah, a half-hour head start. For this first half hour, Tony and Sarah ride at a steady pace of 12 miles per hour. Then, they keep up a steady pace of about 10 miles per hour. When the others start riding, they go at a steady pace of about 15 miles per hour.

a. Write an equation for the distance $d$ in miles Tony and Sarah travel in $t$ hours.

b. Write an equation for the distance $d$ in miles the other riders travel in $t$ hours.

c. Use the equations from parts (a) and (b) to make a table and a graph showing the relationship between distance and time for the two groups of riders.

d. Will the older riders catch up with Tony and Sarah before the end of the 30-mile race? Explain using both the table and the graph.
In this investigation, you used a graphing calculator to help you understand relationships between variables. These questions will help you summarize what you have learned.

Think about your answers to these questions. Discuss your ideas with other students and your teacher. Then write a summary of your findings in your notebook.

1. a. Describe the steps required to make a calculator table that shows how two variables are related.
   b. How can you scan a calculator table to answer questions about a relationship?

2. a. Describe the steps required to make a calculator graph that shows how two variables are related.
   b. How can you trace a calculator graph to answer questions about a relationship?

3. What are the advantages and disadvantages of using tables and graphs to solve problems and search for patterns? In what ways is using a calculator more useful than using paper-and-pencil methods? In what ways is it less useful?
In this unit, you studied some basic ideas of algebra. You learned ways to use those ideas to solve problems about variables and the patterns relating variables. In particular, you studied how to

- recognize situations in which changes in variables are related in useful patterns
- describe patterns of change shown in tables and graphs of data
- construct tables and graphs to display relationships between variables
- use algebraic symbols to write equations relating variables
- use tables, graphs, and equations to solve problems
- use a graphing calculator to construct tables and graphs of relationships and to solve equations

**Use Your Understanding: Algebraic Reasoning**

To test your understanding of algebraic ideas and your skill in using algebraic techniques, consider how algebra is involved in a business we all depend on—the shipping of packages from town to town, across the country, and around the world.
1. A shipping company offers two-day shipping of any package weighing up to 2 pounds for $5 plus $0.01 per mile.

   a. Copy and complete the table.

   **Two-Day Shipping Costs**

<table>
<thead>
<tr>
<th>Distance (mi)</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>1,000</th>
<th>1,500</th>
<th>2,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shipping Cost</td>
<td>$5</td>
<td>$5.01</td>
<td>$5.02</td>
<td>$5.03</td>
<td>$5.04</td>
<td>$5.05</td>
<td>$5.06</td>
<td>$5.07</td>
</tr>
</tbody>
</table>

   b. Describe the pattern by which the shipping cost increases as the shipping distance increases.

   c. Make a graph showing shipping charges for distances from 0 to 2,000 miles. Use appropriate labels and scales on the axes.

   d. Write an equation for the relationship between distance $d$ in miles and shipping cost $c$ in dollars.

   e. Use a graphing calculator and the equation in part (d) to check the graph you made in part (c).

   f. Use the table, graph, or equation to find the cost to ship a 1-pound package 450 miles.

   g. Use the table, graph, or equation to figure out how far you can ship a 2-pound package for $35.

**Explain Your Reasoning**

Answer the following questions in your own words to summarize what you know about variables and patterns.

2. What does the word *variable* mean in algebra?

3. What are *dependent* and *independent variables* and how are they usually related to each other in a problem situation?

4. Give examples that show at least two common patterns that occur in the values of related variables.

5. What are the main decisions and procedures involved in making a table to illustrate the relationship between two variables? How does a table help you describe and study a relationship?
6. What are the main decisions and procedures involved in making a graph to illustrate the relationship between two variables? How does a graph help you describe and study a relationship?

7. What are the main decisions and procedures involved in writing an equation to illustrate the relationship between two variables? How does an equation help you describe and study a relationship?

8. How can you use a graphing calculator to study relationships between variables? What do you need to know about a situation in order to use a calculator? How can the calculator be used to solve problems about variables and relationships?

**Look Ahead**

Your understanding of algebraic ideas and skills will grow as you work on future *Connected Mathematics* units. You will use variables and symbolic expressions in the geometry unit *Stretching and Shrinking*, the proportional-reasoning unit *Comparing and Scaling*, the algebra unit *Moving Straight Ahead*, and all units in the next *Connected Mathematics* course.
change  To become different. For example, temperatures rise and fall, prices increase and decrease, and so on. In mathematics, quantities that change are called *variables*.

**coordinate graph**  A graphical representation of pairs of related numerical values that shows the relationship between two variables. It relates the independent variable (shown on the *x*-axis) and the dependent variable (shown on the *y*-axis).

**coordinate pair**  An ordered pair of numbers used to locate a point on a coordinate grid. The first number in a coordinate pair is the value for the *x*-coordinate, and the second number is the value for the *y*-coordinate. A coordinate pair for the graph shown above is (0, 60).

**dependent variable**  One of the two variables in a relationship. Its value depends upon or is determined by the other variable called the *independent variable*. For example, the cost of a long-distance phone call (dependent variable) depends on how long you talk (independent variable).

**equation, formula**  A rule containing variables that represents a mathematical relationship. An example is the formula for finding the area of a circle: \( A = \pi r^2 \).
**independent variable** One of the two variables in a relationship. Its value determines the value of the other variable called the **dependent variable**. If you organize a bike tour, for example, the number of people who register to go (independent variable) determines the cost for renting bikes (dependent variable).

**pattern** A change that occurs in a predictable way. For example, the squares on a checkerboard form a pattern in which the colors of the squares alternate between red and black. The sequence of square numbers: 1, 4, 9, 16, ... forms a pattern in which the numbers increase by the next odd number. That is, 4 is 3 more than 1, 9 is 5 more than 4, 16 is 7 more than 9, and so on.

**relationship** An association between two or more variables. If one of the variables changes, the other variable may also change, and the change may be predictable.

**rule** A summary of a predictable relationship that tells how to find the value of a variable. A rule may be given in words or as an equation. For example, this rule relates time, rate, and distance: distance is equal to rate times time, or \( d = rt \).

**scale** A labeling scheme used on each of the axes on a coordinate grid.

**table** A list of values for two or more variables that shows the relationship between them. Tables often represent data made from observations, from experiments, or from a series of arithmetic operations. A table may show a pattern of change between two variables that can be used to predict values not in the table.

**variable** A quantity that can change. Letters are often used as symbols to represent variables in rules or equations that describe patterns.

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**variable independiente** Una de las dos variables relacionadas. Su valor determina el de la otra variable, llamada variable dependiente. Por ejemplo, si organizas un recorrido en bicicleta, el número de personas inscritas (variable independiente) determina el costo del alquiler de las bicicletas (variable dependiente).

**patrón** Una variación que ocurre de manera predecible. Por ejemplo, los cuadrados del tablero de damas forman un patrón en el que los colores de los cuadrados se alternan. La secuencia de números cuadrados: 1, 4, 9, 16, ... forma un patrón en el que los números aumentan según la cifra del siguiente número impar. Es decir, 4 es 3 más que 1, 9 es 5 más que 4, 16 es 7 más que 9, y así sucesivamente.

**relación** Una asociación entre dos o más variables. Si una de las variables cambia, la otra variable también puede cambiar, y dicho cambio puede ser predecible.

**regla** Un resumen de una relación predecible que indica cómo hallar el valor de una variable. Se trata de un patrón que, debido a su coherencia, puede escribirse, convertirse en una ecuación, representarse gráficamente o utilizarse para hacer una tabla. Por ejemplo, la siguiente regla relaciona tiempo, velocidad y distancia: la distancia es igual al producto de la velocidad y el tiempo, o sea \( d = rt \).

**tabla** Una lista de valores para dos o más variables que muestra la relación existente entre ellas. Frecuentemente, las tablas contienen datos provenientes de observaciones, experimentos o de una serie de operaciones aritméticas. Una tabla puede mostrar un patrón de variación existente entre dos variables, el cual puede utilizarse para predecir los valores de otras entradas de la tabla.

**variable** Una cantidad que puede cambiar. Suele usarse letras como símbolos para representar las variables de las reglas o ecuaciones que describen patrones.
**x-axis** The number line that is horizontal on a coordinate grid.

**eje de las x** La línea numérica horizontal en una cuadrícula de coordenadas.

**y-axis** The number line that is vertical on a coordinate grid.

**eje de las y** La línea numérica vertical en una cuadrícula de coordenadas.
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